## ELECTRICAL CONDUCTIVITY OF AN ARGON PLASMA

IN A STABILIZED ARC
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A method is described for determining the electrical conductivity $\sigma$ as a function of the temperature T from measurements in extended plasma sources of radial symmetry. The accuracy and features of the method are analyzed in numerical examples. Measurements made for a stabilized argon arc and the $\sigma(\mathrm{T})$ dependence determined from them are the argon plasma are reported. The results are analyzed and compared with theory and other experiments.

Study of many processes occuring in a plasma requires knowledge of the dependence of the electrical conductivity $\sigma$ on the temperature T. Theoretical $\sigma(\mathrm{T})$ dependences based on various equations for the conductivity and on various data for the cross sections for collisions between plasma particles yield very different results [1], so reliable experimental methods for determining the conductivity are important.

It is difficult to determine $\sigma(\mathrm{T})$ because an artificially produced plasma is generally nonisothermal throughout its volume. The quantities and effects associated with the conductivity here are of an integral nature, so it is difficult to interpret experimental results. Steady-state plasmas at $10,000-15,000^{\circ} \mathrm{K}$ and above are usually produced by electric arcs. The methods available for determining $\sigma(\mathrm{T})$ from arc measurements have several disadvantages.

The method based on measurement of the average atomic cross sections $Q_{a}[2,3]$ is based on theoretical equations for the conductivity, requires knowledge of the cross sections for interactions between electrons and ions, and does not take into account the temperature dependence of $Q_{a}$. A method independent of this theory is described in [4], but the basic assumptions behind this method limit its application. For example, this method cannot in principle be used for the case of arcs in intense gas flows, in arcs with an optically opaque plasma, and in certain other particular cases. Two other methods [5, 6] require measurement of the arc parameters under quite a large variety of conditions with a large temperature range at the column axis. However, it is not at all possible to stabilize a plasma temperature over a wide range in all arcs. In addition, these methods are sensitive to errors in the measurement of the arc parameters and have certain other disadvantages.

1. We assume a plasma occupying a rather large volume whose temperature and other properties (the arc column, the plasma stream, etc.) are readily symmetric about the longitudinal $z$ axis. An electric field acts on the plasma along the $z$ direction, causing a current I; some potential distribution $V(z)$ is set up. The electric field intensity $E=d V / d z$ is constant in $z=$ const cross sections. The plasma is at thermodynamic equilibrium, so there is a single-valued dependence between $\sigma$ and $T$ for it. Then for a given cross section $z_{~}=$ const with a radial temperature distribution $T(r)$, we can write Ohm's law as

$$
\begin{equation*}
\frac{I}{E}=G=2 \pi \int_{0}^{R} \sigma[T(r)] r d r \tag{1.1}
\end{equation*}
$$

where $G$ is the integral conductivity over the cross section, and $R$ is the radius of the outer plasma boundary in this cross section.

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Fig. 1


Fig. 2

We assume that we have experimentally determined the integral conductivities $G_{i}=I_{i} / E_{i}$ and temperature profiles $T_{i}(r)$ for some number $N$ of different states (conditions) of the plasma in the cross sections $\mathrm{z}=$ const $(\mathrm{i}=1,2, \ldots, \mathrm{~N})$. We are to determine $\sigma$ as a function of T from these data in the range $\mathrm{T} \leq \max \mathrm{T}(0)\left[\max \mathrm{T}(0)\right.$ is the greatest of the axial temperatures $\left.\mathrm{T}_{\mathrm{i}}(0), \mathrm{i}=1,2, \ldots, \mathrm{~N}\right]$.

We seek the $\sigma(T)$ depenence as some analytic function $\sigma^{\circ}(T)$, whose form may be chosen on the basis of the following, quite obvious considerations.

1. For any maximum plasma temperature $\mathrm{T}_{\max }$ there is always a temperature $\mathrm{T}_{0}<\mathrm{T}_{\max }$ below which we have $\sigma(\mathrm{T}) \ll \sigma\left(\mathrm{T}_{\text {max }}\right)$ or, approximately,

$$
\begin{equation*}
\sigma(T) \approx \sigma_{0}{ }^{\circ}(T)=0 \quad\left(T<T_{0}\right) \tag{1.2}
\end{equation*}
$$

The value of $\mathrm{T}_{0}$ may be called the "relative thermal boundary of the conductivity."
2. In any temperature range ( $\mathrm{T}_{1}, \mathrm{~T}_{2}$ ) which is not too large, the function $\sigma(\mathrm{T})$, which is obviously continuous and smooth, may be described quite accurately by a polynominal of the form

$$
\begin{equation*}
\sigma(T) \approx \sigma_{1}^{\circ}(T)=\sum_{k=0}^{m} a_{k}\left(T-T_{1}\right)^{k} \quad\left(T_{1}<T<T_{2}\right) \tag{1.3}
\end{equation*}
$$

where $a_{k}(k=0.1, \ldots, m)$ are certain coefficients.
The zeroth-order function $\sigma_{0}{ }^{\circ}(\mathrm{T})$ and the polynominal $\sigma_{1}{ }^{\circ}(\mathrm{T})$ may be represented as a single function $\sigma^{\circ}(T)$ which is continuous for all $T<T_{\text {max }}$. For this purpose we must assume $T_{1}=T_{0}, T_{2}=T_{\max }, a_{0}=0$ and, at least, $\mathrm{a}_{1}=0$. Then we can write $\sigma(\mathrm{T})$ dependence in the interval $\mathrm{T} \leq \max \mathrm{T}(0)$ in the form

$$
\begin{array}{cc}
\sigma^{\circ}(T)=\sigma_{0}{ }^{\circ}(T)=0 & \left(T \gtrless T_{0}\right)  \tag{1.4}\\
\sigma^{\circ}(T)=\sigma_{1}{ }^{\circ}(T)=\sum_{k=l}^{m} a_{k}\left(T-T_{0}\right)^{k} & \left(T_{0} \gtrless T<\max T(0)\right)
\end{array}
$$

where $l \geq 2$. The value of $\mathrm{T}_{0}$ along with the coefficients $a_{\mathrm{k}}(\mathrm{k}=l, l+1, \ldots, \mathrm{~m})$ must be treated here as a free parameter of the function $\sigma^{\circ}(T)$. Using (1.4), we express the radial conductivity distributions in terms of the known temperature profiles:

$$
\begin{array}{rc}
\sigma_{i}(r)=\sigma\left[T_{i}(r)\right] \approx \sigma^{o}\left[T_{i}(r)\right]= & \sum_{k=1}^{m} a_{k}\left[T_{i}(r)-T_{0}\right]^{k} \quad\left(0 \gtrless r \gtrless r_{0 i}\right)  \tag{1.5}\\
\sigma_{i}(r) \approx 0 \quad\left(r_{0 i}<r<R_{i}\right)
\end{array}
$$

TABLE 1

| Version | $T^{*}(0),{ }^{\circ} \mathrm{K}$ | ${ }^{*}, \mathrm{mho}$ <br> cm | $N^{*}$ | $m$ | $\left\langle\Delta \sigma^{*}\right\rangle, \mathrm{mho}$ <br> cm | $\left\langle\delta^{*}\right\rangle \cdot 100 \%$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $9220 \div 13470$ | $1.80 \div 12.95$ | 10 | 2 | 1.16 | 2.7 |
| 2 | $9220 \div 13470$ | $\mathbf{1 . 8 0 \div 1 2 . 9 5}$ | 10 | 3 | 0.78 | 1.8 |
| 3 | $9220 \div-13470$ | $1.80 \div 12.95$ | 10 | 4 | 0.52 | 1.2 |
| 4 | $9220 \div 13470$ | $1.80 \div 12.95$ | 10 | 5 | 0.20 | 0.5 |
| 5 | $10290 \div 13470$ | $3.25 \div 12.95$ | 8 | 4 | 0.54 | 1.2 |
| 6 | $9940 \div 13470$ | $2.61 \div 12.95$ | 5 | 4 | 0.52 | 1.2 |
| 7 | $9220 \div 13470$ | $1.80 \div 12.95$ | 4 | 4 | 0.51 | 1.2 |
| 8 | 13500 | $3.72 \div 8.68$ | 4 | 4 | 0.56 | 1.3 |

where $R_{i}$ is the radius of the inner plasma boundary in the i-th state, and $r_{0 i}$ are the $r$ values corresponding to the temperature $T=T_{0}(i=1,2, \ldots, N)$. Substituting (1.5) into (1.1), we find the integral conductivities

$$
\begin{gather*}
G_{i} \approx G_{i}^{\circ}=\sum_{k=l}^{m} a_{k} \Phi_{k i}  \tag{1.6}\\
\Phi_{k i}=2 \pi \int_{0}^{r_{n i}}\left[T_{i}(r)-T_{0}\right]^{k} r d r \quad(i=1,2, \ldots, N ; k=l, l+1, \ldots, m) \tag{1.7}
\end{gather*}
$$

We seek the optimum values of the parameters $a_{k}(k=l, l+1, \ldots, m)$ and $T_{0}$ from the condition for the best fit of the quantities

$$
G_{i}{ }^{\circ}=2 \pi \int_{0}^{R_{i}} \sigma^{\circ}\left[T_{i}(r)\right] r d r
$$

to the actual integral conductivities

$$
G_{i}=2 \pi \int_{0}^{R_{i}} \sigma\left[T_{i}(r)\right] r d r
$$

for a set of all $N$ states $(i=1,2, \ldots, N)$. According to the method of least squares, the best fit of $G_{i}{ }^{\circ}$ and $G_{i}$ occurs when

$$
\begin{equation*}
S=\sum_{i=1}^{N}\left(G_{i}-G_{i}{ }^{\circ}\right)^{2}=\min , \quad \text { or } \quad S=\sum_{i=1}^{\dot{N}}\left(G_{i}-\sum_{k=l}^{m} a_{k} \Phi_{k i}\right)^{2}=\min \tag{1.8}
\end{equation*}
$$

This condition will evidently hold when all the partial derivatives of $S$ with respect to the free parameter vanish:


Fig. 3

$$
\begin{gather*}
\frac{\partial S}{\partial a_{j}}=\frac{\partial}{\partial a_{j}}\left[\sum_{i=1}^{N}\left(G_{i}-\sum_{k=l}^{m} a_{i{ }^{\prime}} \Phi_{k i}\right)^{2}\right]=0 \quad(j=l, l+1, \ldots, m) .  \tag{1.9}\\
d S / d T_{0}=0 \tag{1.10}
\end{gather*}
$$

Expanding the products in (1.9), we find $m-l+1$ equations

$$
\begin{equation*}
\sum_{k=l}^{m} a_{k} \sum_{i=1}^{N} \Phi_{k i} \Phi_{j_{i}}=\sum G_{i} \Phi_{j i} \quad \quad(j=l, l+1, \ldots, m) \tag{1.11}
\end{equation*}
$$

which are linear with respect to the $\mathrm{m}-l+1$ unknowns $a_{\mathrm{k}}(\mathrm{k}=l, l+1$, $\ldots, m$ ) at fixed values of $T_{0}$. When $T_{0}$ is taken into account, the total number of unknowns is $m-l+2$. Equation (1.10) should be considered the missing equation. Under the condition $m-l+2 \leq N$, the optimum parameters $a_{k}$ and $\mathrm{T}_{0}$ may be determined by means of Eqs. (1.11) and (1.10) in the following manner.

Within the region $\mathrm{T}<\min \mathrm{T}(0)$ [i, e., the region below the smallest of all the axial temperatures $\left.\mathrm{T}_{\mathrm{i}}(0), \mathbf{i}=1,2, \ldots, \mathrm{~N}\right]$, a series of


Fig. 4


Fig. 5
values $T_{0}=T_{0}{ }^{\prime}, T_{0}{ }^{\prime \prime}, T_{0}{ }^{\prime \prime \prime}, \ldots$, is specified; then the quantities $\Phi_{k i}=\Phi_{k i}, \Phi_{k i}{ }^{\prime \prime}, \Phi_{k i}{ }^{\prime \prime}, \ldots(i=1,2, \ldots, N$; $\mathrm{k}=l, l+1, \ldots, \mathrm{~m}$ ) are calculated from Eq. (1.7), and systems of equations of the form (1.11) are calculated from these quantities. The solutions of these equations yield the coefficients $a_{\mathrm{k}}=a_{\mathrm{k}}{ }^{\prime}, a_{\mathrm{k}}{ }^{\prime \prime}, a_{\mathrm{k}}{ }^{\prime \prime \prime}$, $\ldots$, then the corresponding values $S=S^{\prime}, S^{\prime \prime}, S^{\prime \prime \prime}, \ldots$, are calculated from Eq. (1.8). This yields the $S\left(T_{0}\right)$ dependence in the form of individual points $S^{\prime}\left(\mathrm{T}_{0}{ }^{\prime}\right), \mathrm{S}^{\prime \prime}\left(\mathrm{T}_{0}{ }^{\prime \prime}\right), \mathrm{S}^{\prime \prime \prime}\left(\mathrm{T}_{0}{ }^{\prime \prime}{ }^{\prime \prime}\right)$, ... . The optimum value of $\mathrm{T}_{0}$ corresponding to Eq. (1.10) is determined from the minimum of the function $\mathrm{S}\left(\mathrm{T}_{0}\right)$. Solution of system (1.11) written for this $\mathrm{T}_{0}$ value yields the unknown coefficients $a_{\mathrm{k}}(\mathrm{k}=l, l+1, \ldots, \mathrm{~m})$. In this same manner, the function $\sigma^{\circ}(T)$ in the form (1.4) is determined which corresponds to condition (1.8) and which apparently best describes (for the given $l$ and $m$ ) the $\sigma(\mathrm{T})$ dependence.

All the numerical calculations (including the optimization of solutions for $\mathrm{T}_{0}$ ) were programed for an M-20 computer. The computer time required for the complete calculation is $5-10 \mathrm{~min}$, depending primarily on the step $\Delta T_{0}$.
2. The function $\sigma^{\circ}(T) \approx \sigma(T)$ is related in a very complicated manner to the initial values $G_{i}$ and $T_{i}(r)$ so it is not possible to carry out an accurate analysis of the accuracy of the method. Through the use of a computer, however, the method can be easily checked for particular examples. For this purpose some function $\sigma^{*}(T)$ dependence, and arbitrary integral conductivities

$$
G_{i}^{*}=2 \pi \int_{0}^{R_{i}{ }^{*}} \sigma^{*}\left[T_{i}^{*}(r)\right] r d r \quad\left(i=1,2, \ldots, N^{*}\right)
$$

are calculated for a certain number $N^{*}$ of specified temperature profiles $\mathrm{T}_{\mathbf{i}}{ }^{*}(\mathrm{r})$.
Treatment of the quantities $\mathrm{G}_{i}{ }^{*}$ and $\mathrm{T}_{\mathrm{i}}{ }^{*}(\mathrm{r})$ determined in this manner yields the best fit function $\sigma^{\circ}$ ( T ) in the form (1.4). Since this function reproduces the original $\sigma^{*}(\mathrm{~T})$ dependence, one can accurately evaluate the accuracy and reliability of the method.

In a numerical check in this manner, we adopted as the arbitrary function $\sigma^{*}(\mathrm{~T})$ the theoretical $\sigma(\mathrm{T})$ dependence for argon calculated from the equations of [3]. The $\mathrm{G}_{\mathrm{i}}{ }^{*}$ values were calculated with an account of the $\sigma^{*}(\mathrm{~T})$ function chosen for two different groups of $\mathrm{T}_{\mathrm{i}}{ }^{*}(\mathrm{r})$ temperature curves. The first of them is a set of real temperature profiles in an argon-arc column with a range $\mathrm{T}^{*}(0)=9220-13,470^{\circ} \mathrm{K}$ (see Sec. 3 and Fig. 3). The second group of $\mathrm{T}_{\mathrm{i}}{ }^{*}(\mathrm{r})$ curves shown in Fig。 1 were calculated from

$$
T^{*}(r)=T^{*}(0)-\left[T^{*}(0)-T_{w}{ }^{*}\right] r^{n} R^{*-n}
$$

Here $\mathrm{T}_{\mathrm{W}}$ * is the nominal temperature of the outer boundary of the plasma (at the wall). The curves in Fig. 1 correspond to $\eta=2,3,4,5$ (at constant $T^{*}(0)=13,500^{\circ} \mathrm{K}, \mathrm{T}_{\mathrm{W}}^{*}=500^{\circ} \mathrm{K}$, and $\mathrm{R}^{*}=0.25 \mathrm{~cm}$ ). In addition to varying the nature of the $T_{i} *(r)$ curves in these calculations, we varied the $m$ values (at $l=$ const $=2$ ) and the number of states $N^{*}$. The fit of $\sigma^{*}(T)$ to $\sigma^{*}(T)$ was characterized by the averaged integral absolute and relative deviations, $\left\langle\Delta \sigma^{*}\right\rangle$ and $\left\langle\delta^{*}\right\rangle$, respectively, calculated from

$$
\begin{gather*}
\left\langle\Delta \sigma^{*}\right\rangle=\frac{1}{T_{2}-T_{1}} \int_{T_{1}}^{T_{2}}\left|\sigma^{* *}(T)-\sigma^{*}(T)\right| d T  \tag{2.I}\\
\left\langle\delta^{*}\right\rangle=\frac{\left\langle\Delta \sigma^{*}\right\rangle}{\left\langle\sigma^{*}\right\rangle}, \quad\left\langle\sigma^{*}\right\rangle=\frac{1}{T_{2}-T_{1}} \int_{T_{1}}^{T_{2}} \sigma^{*}(T) d T \tag{2.2}
\end{gather*}
$$

over the temperature range $8000-13,500^{\circ} \mathrm{K}$.

TABLE 2

| No | $\mathrm{d}, \mathrm{mm}$ | $I, \mathrm{a}$ | $\mathrm{E}, \mathrm{V} / \mathrm{cm}$ | $G, \mathrm{mho} \mathrm{cm}$ | $r_{p}, \mathrm{a}$ | $\frac{I_{p}-I}{I} \cdot 100 \%$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 5 | 33.2 | 7.4 | 4.49 | 31.8 | -4.2 |
| 2 | 5 | 48.9 | 8.3 | 5.89 | 49.5 | +1.2 |
| 3 | 5 | 70.9 | 9.9 | 7.16 | 73.5 | +3.7 |
| 4 | 5 | 137 | 13.5 | 10.15 | 14.4 | +3.9 |
| 5 | 5 | 165.5 | 14.8 | 11.18 | 159.6 | -3.6 |
| 6 | 6 | 79 | 8.3 | 9.5 | 77 | -2.5 |
| 7 | 6 | 110 | 9.7 | 11.34 | 103 | -7.3. |
| 8 | 6 | 201 | 12.9 | 15.6 | 184 | -8.5 |
| 9 | 8 | 40 | 5.2 | 7.7 | 37.5 | -6.2 |
| 10 | 8 | 80 | 6.0 | 13.4 | 76 | -5.0 |
| 11 | 8 | 200 | 9.2 | 21.7 | 206 | +3.0 |

Table 1 shown the results of eight different versions of the calculation. Versions 1-4 correspond to $m=2-5$, respectively, and the data for the ten conventional states with temperature distributions $\mathrm{T}_{\mathrm{i}}{ }^{*}(\mathrm{r})$ according to Fig. 3. In versions 5-7, the number of states $N^{*}$ was changed (from four to eight) at the same $\mathrm{m}=4$. In the eighth version, the function $\sigma^{* \circ}(\mathrm{~T})$ was determined for $\mathrm{m}=4$ from the data of four conventional states with te mperature profiles $T_{i}{ }^{*}(r)$ having identical $T^{*}(0)=13,500^{\circ} \mathrm{K}$ (Fig. 1). The $G_{i}^{*}$ values for the conventional states used are shown in Table 1.

Analysis of the data in the table yields the following conclusions.

1. Versions 1-4 show that the accuracy with which the $\sigma^{*}(\mathrm{~T})$ dependence is reproduced by $\sigma^{* o}(\mathrm{~T})$ increases with increasing $m$ 。When exact $G_{i}$ and $T_{i}(r)$ values are available, one can apparently achieve an arbitrarily accurate determination of $\sigma(\mathrm{T})$ in form (1.4) by increasing $m$ with $l=$ const. Since, however, $G_{i}$ and $T_{i}(x)$ may be determined experimentally within $1 \%$ or a few percent, it is in fact sufficient to use $\mathrm{m}=3-5$ (for $l=2$ )。
2. It follows from versions $3,5,6,7$ that for constant $l$ and $m$ the fit of $\sigma^{*}(\mathrm{~T})$ to $\sigma^{*}(\mathrm{~T})$ is essentially independent of $N^{*}$ (for $N^{*} \geq m-l+2$ ). This means that in determining $\sigma(T)$ by this method it is in principle sufficient to have available $\mathrm{G}_{\mathbf{i}}$ and $\mathrm{T}_{\mathrm{i}}(\mathrm{r})$ for the minimum number of states $\mathrm{N}_{\min }=m-l+2\left(e . g ., \mathrm{N}_{\min }=3\right.$ for $l=2$ and $m=3$ ). On the other hand, the number of states has no upper limit. If there is a sufficiently strong inequality $N>m-l+2$, it follows from Eqs. (1.11) that an averaging of the random measurement errors in $G_{i}$ and $T_{i}(r)$ will automatically occur in the process of determining the $\sigma(\mathrm{T})$ dependence.
3. Use of the $\mathrm{T}_{\mathrm{i}}{ }^{*}$ curves with identical $\mathrm{T}^{*}(0)=13,500^{\circ} \mathrm{K}$ yields essentially the same result as in the case of the $\mathrm{T}_{\mathrm{i}}{ }^{*}(\mathrm{r})$ curves with an interval $\mathrm{T}^{*}(0)=9220-13,470^{\circ} \mathrm{K}$ (cf.version 7 and 8 ). This implies that the temperature interval at the axis is of no fundamental importance in this procedure.
4. The method described above was used to determine the electrical conductivity of an argon plasma at atmospheric pressure and at temperatures up to about $13,500^{\circ} \mathrm{K}$. The experiments were carried out in an arc stabilized by copper diaphragms [7]. The stabilizing arc channel had a diameter of $\mathrm{d}=5 \mathrm{~mm}$ and consisted of several cooled sections (each section was composed of several diaphragms). The cathode and anode parts and the diaphragms near the electrodes were also cooled individually. The number of sections in a channel ranged from two to five, and the total arc length ranged from 4.86 to 12.3 cm . In one of several diaphragms there was a window for optical measurements transverse to the arc column. The test gas (argon) was supplied to the arc from the cathode direction at a constant rate of $r=0.2 \mathrm{~g} / \mathrm{sec}$.

The current $I$, the voltage $U$ across the electrodes, the power $W_{w i}$ transferred from the plasma to the walls in $n$ individual arc regions ( $i=1,2, \ldots, n$ ), the electric field intensity $E$ and the temperature distribution $\mathrm{T}(\mathrm{r})$ in the column were measured for I in the range $5-190 a$.

The power $W_{w i}$ absorbed by the walls was determined by a calorimetric method. Since the argon flow was so slight, the energy carried away from the arc with the gas was negligible [8], and we would expect

$$
\begin{equation*}
\sum_{i=1}^{n} W_{u i} \approx W=I U \tag{3.1}
\end{equation*}
$$

to hold, where $W$ is the arc power. The experimental $I$, $U$, and $W_{W i}(i=1,2, \ldots, n)$ values satisfied Eq. (3.1) within $1-1.5 \%$; this is evidence that the measurements were reliable.


Fig. 6

The field intensity E was determined by two different methods. In the first method, E was found from a treatment of the U(I) characteristics measured for various arc lengths $l$. These characteristics were used to plot the $U(l)$ dependences (for the $I=$ const values of interest), which were supposed to be linear if the are column was cylindrically symmetric and had constant conditions at electrodes for all $l$. In this case, the slope of the $\mathrm{U}(l)$ curves yield the field intensity in the column.

In the second method, we have

$$
\begin{equation*}
\frac{W_{w}}{l_{w}}=I E, \quad \text { or } \quad E=\frac{W_{w}}{l_{w} I} \tag{3.2}
\end{equation*}
$$

from the power balance for a cylindrical are column. Here $W_{W}$ is the power absorbed by the walls over the measuring length $l_{\mathrm{W}}$ of the channel.

In the are studied, the column was uniform along its length, and Eqs. (3.2) were essentially satisfiedat argon flow rates of $g \leq 0.05 \mathrm{~g} / \mathrm{sec}$ [8], so the use of these methods in the case $\mathrm{g}=0.02 \mathrm{~g} / \mathrm{sec}$ is completely justified.

To determine $E$ by the first method, we used the $U(I)$ characteristics measured at $l=4.86,5.60,8.13$, and 12.3 cm (curves $1-4$, respectively, in Fig. 2). The $U(l)$ dependences plotted from the se characteristics for various $I=$ const values turned out to be linear. The field intensities in the column found from the slopes of these dependences turned out to be described by the smooth curve 5 also shown in Fig. 2. Here the points show the $E \times 10$ values found by the second method. Using (3.2), through the use of calorimetric data in various regions (sections) of the stabilizing channel. The two methods agree within the experimental accuracy.

The plasma temperature in the arc column was determined from the absolute intensity of the argon continuous spectrum at $\lambda=4300 \AA$. The continuum luminance was determined from the blackening of photographs of the arc obtained with an ISP-51 spectrograph; the radiation of the anode spot of a carbon arc with a known spectral luminance was used as standard [9]. The intensity profiles observed at the site of the arc were converted into radial radiation-density distributions $\varepsilon_{\lambda}(r)$ through a solution of the integral Abel equation. To determine the temperature profiles $T(r)$ from the measured $\varepsilon_{\lambda}(r)$ dependences, we used the $\varepsilon_{\lambda}(T)$ dependence calculated from the Biberman - Norman theory with an account of the experimental data on argon emission given in [10]. This method yielded temperature distributions in the luminous zone of the column at arc currents in the range $5.6-180 a$.

Before we could use the data from the arc measurements to detexmine $\sigma(\mathrm{T})$, we had to determine the validity of the assumption of a local thermodynamic equilibrium in the plasma column. As a measure of the deviation from equilibrium we adopted the quantity $T_{e}-T_{g}\left(T_{e}\right.$ is the electron temperature, and $T_{g}$ is the temperature of the heavy particles, atoms and ions), using [11]

$$
\begin{equation*}
\frac{T_{e}-T_{g}}{T_{e}}=\frac{m_{g}}{4 m_{e}} \frac{\left(\lambda_{e} e E\right)^{2}}{\left(\sqrt[3]{3} / k T_{e}\right)^{2}} \tag{3.3}
\end{equation*}
$$

for the calculation. Here $m_{g}$ is the mass of the heavy particles, $e$ and $m_{e}$ are the charge and mass of the electron, $k$ is the Boltzmann constant, $\lambda_{e}=1 /\left(n_{a} Q_{a}+n_{i} Q_{i}\right)$ is the mean free path of the electron, and $n_{a}$ and $n_{i}$ are the concentrations of atoms and ions. The cross sections $Q_{a}$ and $Q_{i}$ for collisions between electrons and atoms and ions, respectively, were taken from [3]. The temperature $T_{e}$ was assumed equal to that measured experimentally at the column axis. [The calculations were carried out only for the are axis, where there was no temperature gradient or corresponding heat transfer by the electron gas, not taken into account by Eq. (3.3).]

Large deviations of $\mathrm{T}_{\mathrm{e}}$ from $\mathrm{T}_{\mathrm{g}}$ (i.e., more than $10 \%$ ) were observed at arc currents $\mathrm{I} \lesssim 10 a$. At higher currents ( $\gtrsim 50 a)$, the difference $\mathrm{T}_{\mathrm{e}}-\mathrm{T}_{\mathrm{g}}$ was only $1 \%$ of the measured T . The results of these estimates are in agreement with the experimental data of Kolesnikov [15].

The experimental temperature distribution $T(r)$ in the column are shown in Fig. 3 for ten are states with currents from 11.7 to $180 a$; curves $1-10$ correspond to $I=11.7,18.4,24.9,38.1,60,81.3,99.1,120$,
160.5 , and $180 a$, respectively. The regions of the $T(r)$ curves shown by the solid lines were obtained directely from a treatment of the optical data. The peripheral regions, shown by dashed lines, were obtained by an interpolation between the least measured $T(r)$ values and the wall temperature $T_{W}$. The temperature $\mathrm{T}_{\mathrm{W}}$ was found from a thermal calculation carried out for the diaphragms with an account of the calorimetric data.
4. The argon conductivity $\sigma(T)$ was determined from $G$ and $T(r)$ for these ten arc states (Fig. 3). The integral conductivities $G$ were calculated with an account of the correction for the parasitic current i' flowing through eash diaphragm and partially shunting the arc column (the parasitic currents through the diaphragms were due to the potential difference $\Delta V$ across the column segments spanned by the diaphragms and by the nonideal insulation between diaphragms). This correction was determined from the empirical equation

$$
\begin{equation*}
i^{\prime} \approx 2.5 \cdot 10^{-7} \delta_{d} E E^{2, .86} \tag{4.1}
\end{equation*}
$$

where $\delta_{\mathrm{d}}$ is the diaphragm thickness in centimeters. The G values were calculated from the current $\mathrm{I}^{\prime}=\mathrm{I}-$ $i^{\prime}$, where $I$ is the current measured in the external circuit of the arc. The corrections for the parasitic current were large at high arc currents.

The $\sigma$ on $T$ dependence was sought in form (1.4) for $l=2$ and $m=3$. The problem of determining the parameters $\mathrm{T}_{0}, a_{2}$, and $a_{3}$ by means of Eqs. (1.10) and (1.11) was solved in two ways. First, Eqs. (1.11) were written down and solved for $\mathrm{T}_{0}=1000,2000, \ldots, 8000^{\circ} \mathrm{K}$ and with an account of the $a_{\mathrm{k}}(\mathrm{k}=2.3)$ obtained; each time, the sums $S$ of the squared discrepancies (1.8) were calculated. A rough estimate of the optimum value $\mathrm{T}_{0} \approx 6000^{\circ} \mathrm{K}$ was determined from the minimum of the $\mathrm{S}\left(\mathrm{T}_{0}\right)$ dependence obtained (Fig. 4). Then a more detailed optimization of the solution was carried out over the $\mathrm{T}_{0}$ range $5500-6500^{\circ} \mathrm{K}$ at a step of $\Delta \mathrm{T}_{0}=$ $100^{\circ} \mathrm{K}$. As a result, the optimum parameters $\mathrm{T}_{0}=5900^{\circ} \mathrm{K}, a_{2}=2.66 \cdot 10^{-6} \mathrm{mho} / \mathrm{cm}\left({ }^{\circ} \mathrm{K}\right)^{2}$, and $a_{3}=-0.188 .10^{-9}$ mho/cm $\left({ }^{\circ} \mathrm{K}\right)^{3}$ were found.

In the same manner, the dependence of $\sigma$ on T for argon for $\mathrm{T} \lesssim \max \mathrm{T}(0) \approx 13,500^{\circ} \mathrm{K}$ at atmospheric pressure was determined to be

$$
\sigma(T) \approx\left\{\begin{array}{cl}
0 & \left(T \gtrless 5900^{\circ} \mathrm{K}\right)  \tag{4.2}\\
2.66 \cdot 10^{-8}(T-5900)^{2}-0.188 \cdot 10^{-9}(T-5900)^{3} & \left(5900 \gtrless T \gtrless 13500^{\circ} \mathrm{K}\right)
\end{array}\right.
$$

Curve 1 in Fig. 5 shows the $\sigma(T)$ dependence calculated from this equation. The region of the curve shown by the solid line corresponds to the column temperatures found in the optical measurements; the region shown by the dashed line corresponds to temperatures found by interpolation of the $T(r)$ curves at the channel periphery (Fig. 3). The conductivity $\sigma$ increases from about 12 to $71 \mathrm{mho} / \mathrm{cm}$ for the measured range $\mathrm{T} \approx 8200-13,500^{\circ} \mathrm{K}$.

Several special calculations were carried out to analyze and evaluate the accuracy with which the $\sigma(T)$ dependence was determined. Figure 6 shows the current distribution over the column cross section:

$$
\begin{equation*}
I_{r}(r)=2 \pi E \int_{0}^{r} \sigma[T(r)] r d r \tag{4.3}
\end{equation*}
$$

calculated using Eq. (4.2) and the experimental $E, T(r)$, and $i^{\prime}$ values for five are states at currents $I=18.4$, $38.1,81.3,120,180 a$ (curves $1-5$, respectively). It follows from these calculations that the central regions of the column, where the $T(r)$ values were obtained directly from their optical measurements, make the primary contribution to the arc current. On the average, about $15 \%$ of the total current corresponds to the peripheral zones, where the temperatures were found by interpolation. In the same manner, the errors associated with the interpolation do not cause significant errors in the $\sigma$ determination over the temperature range studied.

This was confirmed by other calculations: the $\sigma(\mathrm{T})$ values obtained by a treatment of the same initial data, but with a different (clearly implausible) imterpolation of the $T(r)$ curves at the periphery, differ from (4.2) by a few percent (for $T \approx 8200-13,500^{\circ} \mathrm{K}$ ). When the maximum errors in the measurement of $\mathrm{I}, \mathrm{E}$, and $T(r)$ are taken into account, the resultant error in the $\sigma(T)$ determination is, according to numerical estimates, about $\pm 15 \%$ (an average for the range $T \approx 8200-13,500^{\circ} \mathrm{K}$ ).

The reliability of the $\sigma(\mathrm{T})$ dependence determined was checked indirectly through comparison of the calculated arc currents:

$$
I_{p}=2 \pi E \int_{0}^{R} \sigma[T(r)] r d r
$$

with the measured $I$. This comparison was carried out for five $d=5 \mathrm{~min}$ arc states which were studied but not used for the $\sigma$ determination and for six states of analogous argon ares with channel diameters $d$ of 6 and 8 mm , studied in [12]; the results of these calculations are shown in Table 2. The calculated and measured currents agree within a few percent. Since the total error in the measurement of the parameters $I, E$, and $T(r)$ may reach a few percent, this agreement between $I_{p}$ and $I$ must be acknowledged to be satisfactory.

In Fig. 5, the $\sigma(T)$ dependence found is compared with theoretical and experimental data available on the argon conductivity. The theoretical $\sigma(\mathrm{T})$ dependence $[12,3]$ is shown by curve 2 . In the range $\mathrm{T} \vDash$ $11,000^{\circ} \mathrm{K}$, the experimental curve (1) and theoretical curve (2) essentially coincide. For $\mathrm{T}>11,000^{\circ} \mathrm{K}$, the theoretical curve is much steeper. Its greatest deviation from experimental curve (1) (at $T \approx 13,500^{\circ} \mathrm{K}$ ) is about $+20 \%$, which is not much greater than the experimental error. Curve 3 shows the experimental $\sigma(T)$ dependence found in [13] from argon-arc measurements by the method described in [4]. This dependence lies about $6-11 \%$ below curve 1 , lying essentially within the error of the $\sigma(T)$ determination in this study. Figure 5 also shows the results of shock-tube measurements of $\sigma$ (circles), obtained in [14]; they are in satisfactory agreement with curve 1. The deviation of the theoretical dependence (2) from the experimental dependence in the range $T ; 11,000^{\circ} \mathrm{K}$ is apparently due to the use in [12] of ion cross sections $Q_{i}$ slightly on the low side [3].

These calculations and comparisons show that the $\sigma(T)$ dependence for argon has been determined quite reliably. The most convincing evidence of this comes from the good agreement between the currents $I_{p}$ and I for markedly different states of independently studied arcs (Table 2) and the satisfactory agreement between results obtained by independent methods with different plasma sources (in this study and in [14]). All this implies that this method of determining plasma conductivity in quite reliable and may be recommended for use in those cases in which the plasma conductivity has not been studied throughly.

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